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ALTERNATIVES TO RETALIATION IN RESPONSE TO STATE SPONSORED TERRORIST ATTACKS

by

Paul James Evans

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Thesis Advisor:	Guillermo Owen
Second Reader:	Jeffrey Kline

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14. ABSTRACT

We consider a game played between a state sponsor of international terrorism, a terrorist organization and the victim of a terrorist attack. The state sponsor wishes to inflict as much damage to the victim as possible without risking retaliation. The victim state wishes to end these attacks as soon as possible, through non-retaliatory means if possible in order to avoid the penalty associated with retaliation. In this thesis we compare and contrast the victim strategies of buyout, political attrition, and espionage tactics in an effort to maximize the profit of the victim and end the game without retaliation.

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ALTERNATIVES TO RETALIATION IN RESPONSE TO STATE SPONSORED TERRORIST ATTACKS

Paul James Evans
Ensign, United States Navy
B.S., United States Naval Academy, 2013

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June 2014

Author: Paul James Evans

Approved by: Guillermo Owen
Thesis Advisor

Jeffrey Kline
Second Reader

Carlos Borges
Chair, Department of Applied Mathematics

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We consider a game played between a state sponsor of international terrorism, a terrorist organization and the victim of a terrorist attack. The state sponsor wishes to inflict as much damage to the victim as possible without risking retaliation. The victim state wishes to end these attacks as soon as possible, through non-retaliatory means if possible in order to avoid the penalty associated with retaliation. In this thesis we compare and contrast the victim strategies of buyout, political attrition, and espionage tactics in an effort to maximize the profit of the victim and end the game without retaliation.

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List of Acronyms and Abbreviations

H sponsor state
LOOP Louisiana Offshore Oil Port
NPS Naval Postgraduate School
T terrorist organization
U.S. United States
USG United States Government
V victim state

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CHAPTER 1:
Introduction

The majority of relevant terrorist organizations of the modern age have a state sponsor. The benefits of these alliance are well understood and tend to be mutually beneficial. For the terrorists, the alliance acts as a force multiplier, drastically increasing the effectiveness of their operations. The state can have varying levels of support ranging from amnesty to providing manpower, funding, and shelter. The state in turn gains a political tool that can act in their interests while maintaining plausible deniablity. The alliances can be characterized as *secret-coalitions*, where the nature, significance and sometimes even the existence of the relationship is unknown to anyone except the sponsor state and the terrorist organization.

State-terrorist coalitions are present with the maritime domain as well. Smaller states can use terrorism as a tool to control sea traffic or inflict economic harm on a target nation. According to the International Maritime Organization, over 90 percent of the world's trade is carried by sea, because it is the most cost-effective and efficient means of mass transit [7]. Freight ships can carry hundreds of 40-foot containers and tankers carry thousands of barrels of oil, gas or chemicals. These deep draft ships travel along well known commercial sea lanes and traverse several geographical choke points that are easily exploitable. Also, the Internet has made targeting these ships even easier, because a terrorist can easily look up a ship's route, nation of origin, cargo, and dates of travel [1].

Another potential target is maritime infrastructure, such as, oil platforms, pipelines, communication cables, bridges, and underwater tunnels. These structures provide transit and communication services, but are also easy targets for a potential terrorist attack. Once again the Internet can provide a swathe of information on potential targets including frequency of use and inspections, cost, and location. A perfect example of this is the Louisiana Offshore Oil Port (LOOP). LOOP is an offshore oil pipeline that stands in 110 feet of water designed so that deep draft tankers can offload their cargo. LOOP has a throughput capacity of 1.7 million barrels of oil per day, handles 13 percent of the U.S.'s crude oil and connects to several Midwest refineries [2] [10]. As is readily apparent, LOOP is an area of potential exploitation of a small nation whose primary export is oil.

1.1 The Problem

Victims of a terrorist attack struggle to neutralize terrorist groups due to their size, informal structure, and mobility. When involved in a state-terrorist coalition, these difficulties are compounded by the sponsor's ability to provide cover and protection within their borders. The coalition makes the terrorist organizations exceptionally hard to find, track, and eliminate, and thusly, hard to deter. International giants as a result are vulnerable to attacks from these covert groups as they cannot bring their economic and militaristic strength to bear on their assailants [3]. The overall advantage in these altercations is generally given to the terrorists. Though they lack the ability to strike decisive blows, their maneuverability and covertness prevents significant retaliatory blows.

Terrorism is an effective tool for weaker nations. The states that sponsor terrorism know that they are no match for their targets, however, they often believe that they have no effective alternative. Strong states such as the U.S., Israel, and India could easily topple Iran, Syria and Pakistan, respectively, however, the strength of these nations fails to deliver a decisive blow to the terrorism that afflicts them. Terrorism is "war by other means" and allows the sponsor to operate in ways they previously could not. As a result, coercing a sponsor to abandon their support is no simple task. Sponsors often go into the coalition understanding the risks that they are taking, however, they also know that a coalition is difficult to spot and therefore difficult to retaliate against. Also, their interests and ideologies often fall in line with that of the terrorists making support even more appealing.

Should a victim believe that a coalition exists, the next challenge that the victim is presented with is how to retaliate. As discussed it is difficult to track and eliminate a terrorist group, therefore direct retaliation is highly unlikely. The alternative is to retaliate indirectly by retaliating against the sponsor physically or through sanctions. The challenge with this form of indirect retaliation is the international repercussions that the victim has to face. Should the victim retaliate without sufficient evidence, the international community will punish it. As a result, the victim is forced to endure several attacks, until it is readily apparent that the terrorists that are responsible should be incapable of attacking to such a degree.

This thesis consists of five additional sections. In Chapter 2, we review the 2011 work by Lindner et al. In Chapter 3, we will discuss a *buyout* strategy for the victim. In Chapter 4, we will analyze the use of an economic exploit by victim against the state sponsor. In Chapter 5, we will analyze a potential use of intelligence against the sponsor state. Finally, we will conclude

in Chapter 6 by summarizing the strengths and weaknesses of of the methods described in Chapters 3,4, and 5.

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CHAPTER 2:
The model

We will continue the game proposed by Lindner [6]. Consider a three-person game between a sponsor/host state (H), the sponsored terrorist organization (T) and the victim of T's attacks (V). Assume V has been attacked by T, however, V is unable to directly retaliate against T. While acting alone, T operates at a very low level and is only truly effective while supported by H. As a result, V will retaliate indirectly against T by attacking or otherwise sanctioning H. The level of support given by H is not readily apparent. Even if V is believes that H is supporting T, there are often political restraints that prevent an attack on H without conclusive evidence. We assume that external parties, such as, neutral countries or international coalitions, will sanction V if V attacks H without sufficient evidence to demonstrate an alliance between H and T. In the absence of physical evidence that definitively links H to T, mathematical analysis may infer a connection. If the magnitude and frequency of T's attacks against V suggest that T cannot be operating by itself, and suggests that a relationship between H and T exists, then neutral parties will accept the existence of a relationship.

In our model we will assume that T is receiving aid from H, but the alliance is unknown to V until V is attacked. While acting alone, the strength of T's attacks is q, where $0 \leq q < 1$. While operating with H, this raises to p, where $q \leq p \leq 1$ and $p = q$ means H does not support T. We will assume that as long as T has support T will continue to attack with strength p. Since there are no strategic considerations for T we will consider only the two person game between H and V.

In addition to the strength of the attacks p, H must also decide at what time $s > 0$ to sever relations with T. At some point in time the game ends because H is no longer allied with T or V retaliates. We assume V does not retaliate until $t \geq 0$. V only retaliates against H if H is still allied with T, so $t < s$. However, if H has abandoned the coalition prior to V's retaliation $s \leq t$. Thus the game ends at $\tau = \min\{s,t\}$. Let the payoffs of V and H be denoted by $\Pi_V(p,\tau)$ and $\Pi_H(p,\tau)$. The profits $\Pi_V(p,\tau)$ consists of the α-discounted cost of being attacked $F^{\alpha}(p,\tau)$ and the expected cost of retaliation $c(p,\tau)$. So

$$\Pi_V(p,\tau) = -F^{\alpha}(p,\tau) - \delta c(p,\tau) \tag{2.1}$$

where

$$\delta = \begin{cases} 1 & \text{if } t < s \\ 0 & \text{otherwise.} \end{cases} \tag{2.2}$$

H receives profits equal to the *beta*-discounted damage done to V, $F^{\beta}(p,\tau)$, and three types of cost:

- $O^1(p,\tau)$: *material* cost of supporting T,
- $O^2(p,\tau)$: accrued *political* costs created by being suspected of supporting T,
- $O^3(p,\tau)$: the costs of *retaliation* by V.

Thus we have

$$\Pi_H(p,\tau) = F^{\beta}(p,\tau) - O^1(p,\tau) - O^2(p,\tau) - \delta O^3(p,\tau) \tag{2.3}$$

We will also assume that all the terms are differentiable with respect to p and τ and in particular $\frac{\partial}{\partial p}O^i(p,\tau) > 0$ and $\frac{\partial}{\partial \tau}O^i(p,\tau) > 0$ for $i = 1,2$, and $\frac{\partial}{\partial \tau}O^3(p,\tau) > 0$ by discounting future payoffs. In conclusion, we have created a *game* with the following considerations

$$\begin{aligned} &\tau = \min\{s,t\} \\ &\Pi_V(p,\tau) = -F^{\alpha}(p,\tau) - \delta c(p,\tau) \\ &\Pi_H(p,\tau) = F^{\beta}(p,\tau) - O^1(p,\tau) - O^2(p,\tau) - \delta O^3(p,\tau). \end{aligned} \tag{2.4}$$

2.1 The Victim's Payoff

Here we will expand upon the payoff $\Pi_V(p,\tau) = -F^{\alpha}(p,\tau) - \delta c(p,\tau)$ of the victim. We assume a cost of Ap per time period, where A is the payoff obtained from a unit level attack of strength p. However since T can execute a attack equal to Aq without support, the net cost from the alliance with H is $A(p-q)$ per time period. There is also a discount factor α. Payoff or benefits of size z, obtained τ time periods in the future, in this case, have a discounted present value $z\mathrm{e}^{-\alpha\tau}$. For a given level p, then, the total discounted costs of attacks to V from time 0 to time τ is

$$F^{\alpha}(p,\tau) = \frac{A(p-q)}{\alpha}(1 - \mathrm{e}^{-\alpha\tau}). \tag{2.5}$$

For V, there is a suspicion factor, μ, which corresponds to neutral observers confidence in the evidence of a coalition between T and H. We assume that $\frac{\mu}{\mu+1}$ of the neutral parties will approve of the attack while $\frac{1}{\mu+1}$ disapprove. At the beginning of the game, μ has a relatively low value $\mu(0) = c$. As long as H is supporting T and T continues to attack at a higher level than q, μ

will increase at what will assume is an exponential rate, $\mu(\tau) = ce^{\lambda\tau}$. Using Bayesian decision making, we find that

$$\lambda = p \ln\left(\frac{p}{q}\right) + (1-p)\ln\left(\frac{1-p}{1-q}\right) \tag{2.6}$$

(See [6] for justification), and thus equals zero when $p = q$.

We now consider the cost of retaliation, $c(p, \tau)$. We assume that if V is going to retaliate, then V remains as an observer until time $\tau = t > s$. At time $\tau = t + \varepsilon$ (where ε is infinitesimally small) if evidence of the H-T coalition exists, then V retaliates. At this point the probability of a positive benefit B is $\frac{\mu}{1+\mu}$, and simultaneously the probability of a political cost K is $\frac{1}{1+\mu}$. The expected, discounted value of this is

$$c(p, \tau) = e^{-\alpha\tau} \frac{K - \mu B}{1+\mu}. \tag{2.7}$$

V's challenge is to determine if and when he should retaliate against H, which it decides by it choice of t.

2.2 Victim's Response

V's maintains a more or less reactionary role within the game. As a rational player V's objective is to maximize his profit even in the worst case scenario. H intends to inflict the greatest possible harm to V, and thus, H will act in a manner that gives V the lowest payoff at any given time. V needs to maximize this lowest payoff in order to ensure consistently better payoffs, i.e. use maximin strategies. It is useful to view Π_V graphically to understand its behaviors, see two example cases in Figure 2.1 and Figure 2.2. These figures demonstrate two key values for V, namely t^* time where maximum payoff with retaliation occurs and $\bar{t}$ the value where the payoff of retaliation exceeds that of no retaliation.

In [6], its is shown that t^* is determined by the positive root of μ^* in the quadratic equation that results from differentiating $\Pi_V(t)|_{\delta=1}$ with respect to t. Namely,

$$-[A(p-q) + \alpha B]\mu^2 + [-2A(p-q) - (\alpha - \lambda)B + (\lambda + \alpha)K]\mu + [\alpha K - A(p-q)] = 0. \tag{2.8}$$

Solving this equation we find that t^* is equivalent to

$$t^* = \begin{cases} \frac{\ln(\mu^*/c)}{\lambda} & \text{if } \mu^* \geq c, \\ 0 & \text{otherwise.} \end{cases} \tag{2.9}$$

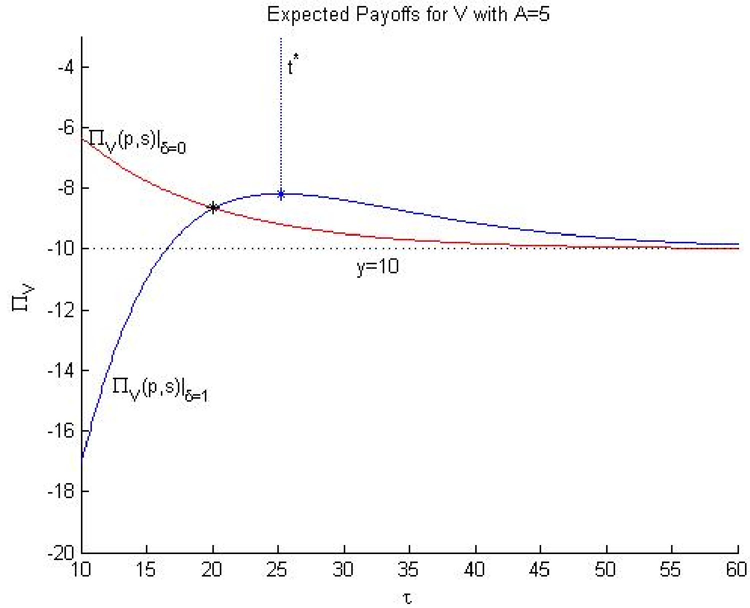

Figure 2.1: In the figure above, the red line depicts V's payoff without retaliation and while the blue is with retaliation. The doted vertical line t^* is the maximum expected payoff if V decides to retaliate. As τ approaches infinity, both payoff functions converge to ten, represented by the asymptote. Graph was generated using the following values $p = .5$, $q = .3$, $A = 5$, $\lambda = .08$, $c = .4$, $\alpha = .1$, $K = 100$, and $B = 50$.

We will call this strategy the *engage strategy*.

$\bar{t}$ is considered the break-even point and occurs when the suspicion factor has grown to a sufficient level that V profits from retaliation as opposed to loosing. Thus,

$$\frac{K - \mu(\bar{t})B}{\mu(\bar{t}+1)} = 0 \implies \mu(\bar{t}) = K/B \tag{2.10}$$

and as such

$$\bar{t} = \frac{\ln(K/Bc)}{\lambda}. \tag{2.11}$$

This break-even point exists in all applications of V's payoff function and is unique. This we will call the *break-even strategy*.

As can be seen in the figures, for low values of A, t^* occurs to the right of $\bar{t}$, Figure 2.1. In this case it V will utilize the *engage strategy*, set $t = \bar{t}$, and retaliate at $t = \bar{t} + \varepsilon$ where ε is infinitesimally small. The *engage strategy* ensures that V will at least $\Pi_V(p,\bar{t})$ which is the maximum value of lower of the curves.

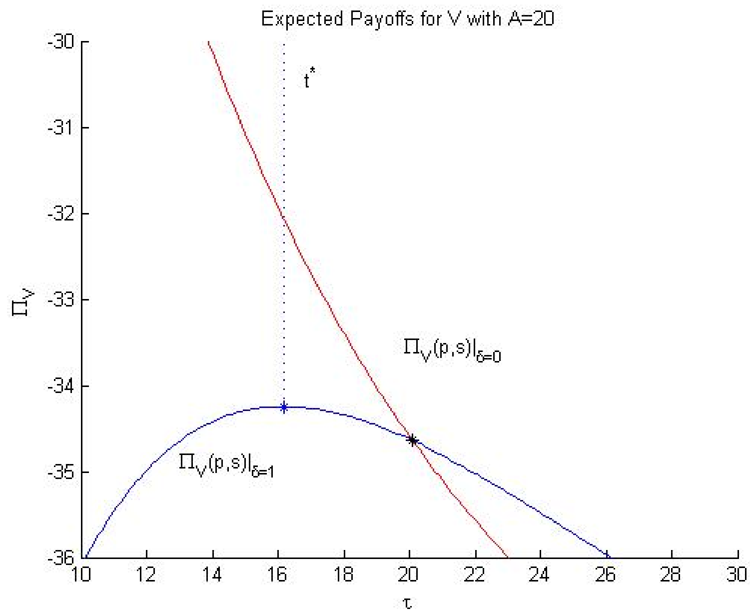

Figure 2.2: In the figure above, the red line depicts V's payoff without retaliation and while the blue is with retaliation. The doted vertical line t^* is the maximum expected payoff if V decides to retaliate. Graph was generated using the following values $p = .5$, $q = .3$, $A = 20$, $\lambda = .08$, $c = .4$, $\alpha = .1$, $K = 100$, and $B = 50$.

Alternatively, for large A, $\bar{t}$ occurs to the right of t^*, Figure 2.2. Now V initiates the *break-even strategy*, setting $t = t^* + \varepsilon$. In this scenario, V understands retaliation at this point will guarantee a payoff equal to the maximum value possible with retaliation.

The only other strategy that V would employ is immediate retaliation, i.e. $t = 0$. This is an unlikely situation, however it is possible if both t^* and $\bar{t}$ exist to the left of $t = 0$. In this case both payoff curves would be monotonically decreasing for all positive t and the best result would be to end the game immediately. Tabularly, the possible payoffs for V can be seen in Table 2.1.

The latest that V will act in any situation is $\max\{0, \bar{t}\}$. It is possible for V to retaliate prior to $\bar{t}$, however, provided the negative repercussions of doing so are small compared to continual attacks. This can occur if the values of B and K are sufficiently small when compared to continual attacks with strength $A(p-q)$. In such a case, a preemptive strike would be favorable to continuous harassment.

Also, for very large $A(p-q)$, the first term in 2.8 will shift the positive root μ^* toward zero.

Table 2.1: V's Maximin Strategies

Order	V's Maximum Strategy	Payoff
$0 \leq t^* \leq \bar{t}$	t^*	$-F(p,t^*) - c(p,t^*)$
$0 \leq \bar{t} \leq t^*$	$\bar{t}$	$-F(p,\bar{t})$
$\bar{t} \leq 0 \leq t^*$	0	0
$t^* \leq 0 \leq \bar{t}$	0	$(cB-K)/(c+1)$
$\bar{t} \leq t^* \leq 0$	0	0
$t^* \leq \bar{t} \leq 0$	0	0

Realistically, this means that the attacks by T are sufficiently strong that V will retaliate against H earlier, regardless of the suspicion factor, in order to prevent future strong attacks.

2.3 The Sponsor State's Payoff

Now we focus on the payoff of H,

$$\Pi_H(p,\tau) = F^\beta(p,\tau) - O^1(p,\tau) - O^2(p,\tau) - \delta O^3(p,\tau), \tag{2.12}$$

where

$$F^\beta(p,\tau) = \frac{A(p-q)}{\beta}(1-\mathrm{e}^{-\beta\tau}) \tag{2.13}$$

indicates the accumulated costs of attacks to V with the discount factor β.

The first cost of H is the *material* cost associated with supporting T. This can be thought of as a correction factor to the gross factor $F^\beta(p,\tau)$. Since H has to payout a cost $C_T > 0$ generated from a unit level attack. Thus, the material cost per time period is $C_T(p-q)$. The total discounted value of these costs at time τ for a given p is

$$O^1(p,\tau) = \frac{C_T(p-q)}{\beta}(1-\mathrm{e}^{-\beta\tau}). \tag{2.14}$$

The political costs associated with supporting T are more complicated. As long as a suspicion of coalition exists, H suffers a loss proportional to the suspicion factor $\frac{\mu(\tau)}{\mu(\tau)+1}$ multiplied by the cost incurred when the suspicion factor reaches its maximum of one. Let $C_P > 0$ represent the political costs associated with a unit level attack. Thus, the cost per time period when the suspicion factor is one is $C_P(p-q)$. Since the political costs accrue over time, they must be

integrated. After being discounted we have

$$O^2(p,\tau) = \int_0^\tau C_P \frac{\mu(v)}{\mu(v)+1}(p-q)\mathrm{e}^{-\beta v}dv. \tag{2.15}$$

Similar to V, the retaliation costs of the H-T alliance with discounted future costs is

$$O^3(\tau) = C_R \mathrm{e}^{-\beta\tau}, \tag{2.16}$$

with $C_R > 0$ (Note O^3 is not p dependent).

2.4 Sponsor State's Behavior

Unlike V, H has the ability to influence the game in two ways, by the level of p and the length of support, s. H can adjust these variables to change his desired payoff, Π_H, however H must also consider when it no longer in his interest to support T. In order understand H's behavior, we must weight his costs, namely

$$\Theta = \frac{A - C_T}{C_P}, \tag{2.17}$$

see Appendix A for derivation. Θ is vital in the selection of s and p by H. In [6] it was shown that H's optimal selection strategies are

$$s^*(t) = \begin{cases} t & \text{for } \Theta \geq 1, \\ \min(s^\#, t) & \text{for } \frac{c}{c+1} < \Theta < 1 \\ 0 & \text{otherwise,} \end{cases} \tag{2.18}$$

where $s^\#$ is the solution to $\frac{\mu(s^\#)}{\mu(s^\#)+1} = \Theta$. The interpretation of this is that for $\Theta > 1$, the terror profit $A - C_T$ outweighs the political costs C_P of an attack. As long as this is the case, then H is satisfied the alliance. Alternatively, if the political costs begin to exceed the terror profit, then H uses a minimum threshold value of Θ as an indicator that it is time to withdrawal. Should the minimum threshold fail to even be reached, H will not support T, because he cannot profit from such an alliance.

Also, Θ governs the selection of p, namely,

$$p^*(t) = \begin{cases} \in (q,1) & \text{for } \Theta \geq \frac{c}{c+1}, \\ q & \text{otherwise.} \end{cases} \tag{2.19}$$

Once again the minimum threshold must be met for H to aid T. Because p is only regarded as the assistance provided to T, as long as Θ remains above the initial suspicion values, T will provide p. It must be noted though that λ and by association μ are functions of p. For $p >> q$, λ is large and thus μ grows quickly and the game is expected to end sooner, however, during this time T inflicts heavy damage to V per attack. Alternatively, for small p, attacks by T are fairly weak, but the suspicion increases so slowly that T can inflict a lot of damage over along time period. Thus, H must decide whether it prefers very few, strong attacks, or weak attacks over a long interval. We will not discuss p in depth here, because we are focusing on alternatives for V, and V cannot influence p.

CHAPTER 3:
Buying and Alliance

In [6], it was suggested that perhaps V could encourage H to dissolve the secret coalition with T. It was suggested that V could affect the effectiveness of a unit level attack by T ($A(x)$), material costs ($C_T(y)$), and political costs ($C_P(z)$) with monetary investments x, y,and z. Through analysis and logical assumptions, it was shown that z was the investment with the greatest return (See Appendix B). In this section we will some implements potential investment methods and their effects on the payoffs of H and V.

3.1 Buyout

Consider first that the victim decides that the best course of action is to attempt to buyout the sponsor state (H). Essentially the victim has a suspicion that a coalition exists between H and T, but does not have sufficient evidence to retaliate without reprisal. From core theory, we understand that the coalition between H and T will remain in effect until such a time that the payoff for H with T is less than that of H with V (we will refer to these as $\Pi_{H,T}$ and $\Pi_{H,V}$ respectively).

3.1.1 Sponsor State's Payoff

V in this scenario intends to payoff H with a single payment $z(\tau)$. We will assume that $z(\tau)$ is a positive decreasing function for all τ, that diminishes after each successful attack. We assume that it is decreasing, because as T's attacks continue V's cost of retaliation is discounted, therefore rather than invest additional money, V will retaliate.

If H decides that he does not want to accept V's offer the game remains the same. In this case, H's payoff would be

$$\Pi_{H,T}(p,\tau) = \frac{(A-C_T)(p-q)}{\beta}(1-\mathrm{e}^{-\beta\tau}) - \int_0^\tau \frac{C_p\mu(v)}{\mu(v)+1}(p-q)\mathrm{e}^{-\beta v}dv - \delta C_R \mathrm{e}^{-\beta\tau}, \quad (3.1)$$

where $\delta = 1$ if V decides to retaliate and $\delta = 0$, otherwise and $\mu = c\mathrm{e}^{\lambda\tau}$. If V does decide to retaliate, then the game is over, otherwise the game continues until H breaks relations with T.

However, if the H accepts V's offer then H's payoff would be

$$\Pi_{H,V}(p,\tau^{\#}) = \frac{(A-C_T)(p-q)}{\beta}(1-\mathrm{e}^{-\beta\tau^{\#}}) - \int_0^{\tau^{\#}} \frac{C_p\mu(v)}{\mu(v)+1}(p-q)\mathrm{e}^{-\beta v}dv + z(\tau^{\#}), \tag{3.2}$$

where $\tau^{\#}$ is the time when the agreement is reached.

It can be shown that for any τ, $\Pi_{H,V}(p,\tau) > \Pi_{H,T}(p,\tau)$ (See Appendix C), however, H knows that he can achieve a payoff equal to $\max(\Pi_{H,T}(p,\tau))$ as long as H remains with T. This gives us a lower bound for $z(\tau^{\#})$, since $\Pi_{H,V}(p,\tau^{\#}) > \Pi_{H,T}(p,\tau)$ for H to justify breaking his coalition with T. Thus we have,

$$z(\tau^{\#}) > \max(\Pi_{H,T}) - \left(\frac{(A-C_T)(p-q)}{\beta}(1-\mathrm{e}^{-\beta\tau^{\#}}) - \int_0^{\tau^{\#}} \frac{C_p\mu(v)}{\mu(v)+1}(p-q)\mathrm{e}^{-\beta v}dv\right) > 0. \tag{3.3}$$

For ease of notation later we will call this bound $B_H(\tau^{\#})$, so $z(\tau^{\#}) > B_H(\tau^{\#})$.

3.1.2 Victim's Payoff

Now we consider the victim's returns during this scenario. If H has decided to reject V's offer then V expects

$$\Pi_V(p,\tau) = -\frac{A(p-q)}{\alpha}(1-\mathrm{e}^{-\alpha\tau}) - \delta\mathrm{e}^{-\alpha\tau}\frac{(K-\mu B)}{1+\mu}, \tag{3.4}$$

where δ is the same as above. Should H accept the offer, V expects

$$\Pi_{V,H}(p,\tau^{\#}) = -\frac{A(p-q)}{\alpha}(1-\mathrm{e}^{-\alpha\tau^{\#}}) - z(\tau^{\#}), \tag{3.5}$$

where $\tau^{\#}$ is the time when the agreement is reached. Since V is assumed to be a logical player, it follows that V will structure $z(\tau)$ in such a way that $\Pi_{V,H}(p,\tau^{\#}) > \max(\Pi_V(p,\tau))$ (Note that $\max(\Pi_V) < 0$). To do otherwise would not benefit V. So we have that

$$z(\tau^{\#}) < -\max(\Pi_V) - \frac{A(p-q)}{\alpha}(1-\mathrm{e}^{-\alpha\tau^{\#}}), \tag{3.6}$$

which we will call $B_V(\tau^{\#})$. By combining 3.3 and 3.6 we find that

$$B_V(\tau^{\#}) > z(\tau^{\#}) > B_H(\tau^{\#}) > 0. \tag{3.7}$$

These bounds do not hold for all τ, however. Figure 3.1 demonstrates a region where $z(\tau^{\#})$ meets the criteria we set forth in 3.7 and a region where the bounds do not hold. To the right of the intersection of B_V and B_H, the payoff $\Pi_{H,V}$ is no longer greater than $\Pi_{H,T}$ and as such H would remain with T.

The utility in this scenario comes from the lack of committed investment by V. Because, z is a one time payoff that only occurs when H accepts the offer, the game returns to the basic model described in Chapter 2 if the buyout is refused.

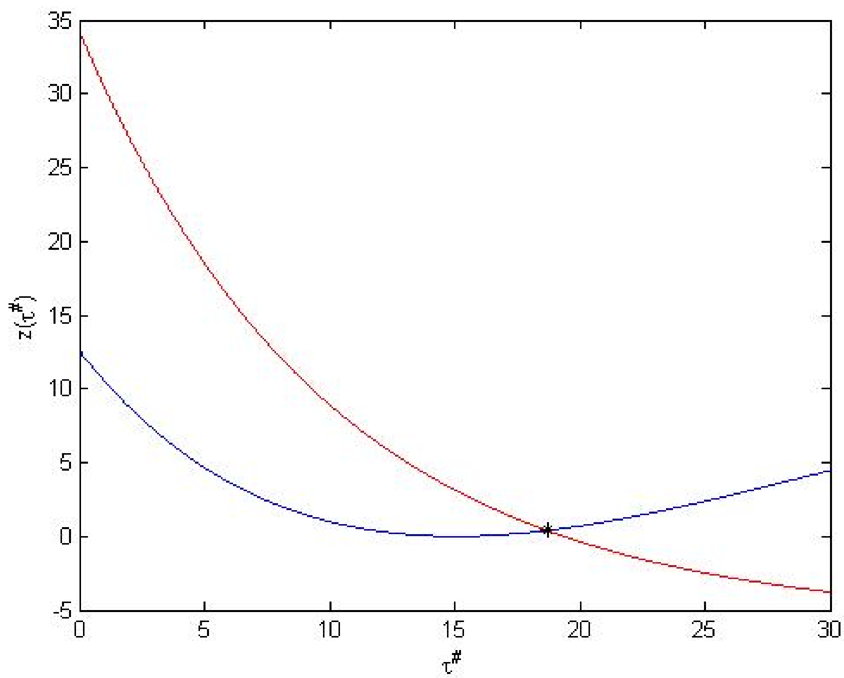

Figure 3.1: In the figure above, the red line depicts $z(\tau^{\#}) = B_V$, and the blue line depicts $z(\tau^{\#}) = B_H$. The region between the two lines and left of the black point, is the region where $B_V > z(\tau^{\#}) > B_H$. Graph was generated using the following values $p = .5$, $q = .3$, $C_P = 15$, $C_T = 5$, $A = 20$, $\lambda = .08$, $c = .4$, $\beta = .1$, $\alpha = .1$, $K = 100$, and $B = 50$.

Since, V is trying to maximize his payoff, he is going to try to minimize z, but his offered payoff to H must exceed H's current payoff with T. Therefore, V will always pick z as close to the lower bound as possible. Figure 3.2 demonstrates V's payoff with $z(\tau^{\#}) = B_H(\tau^{\#})$, and compares the buyout strategy to the basic model's options.

As can be seen in the Figure 3.2, there exists a time t_{sell} at which point the buyout is no longer

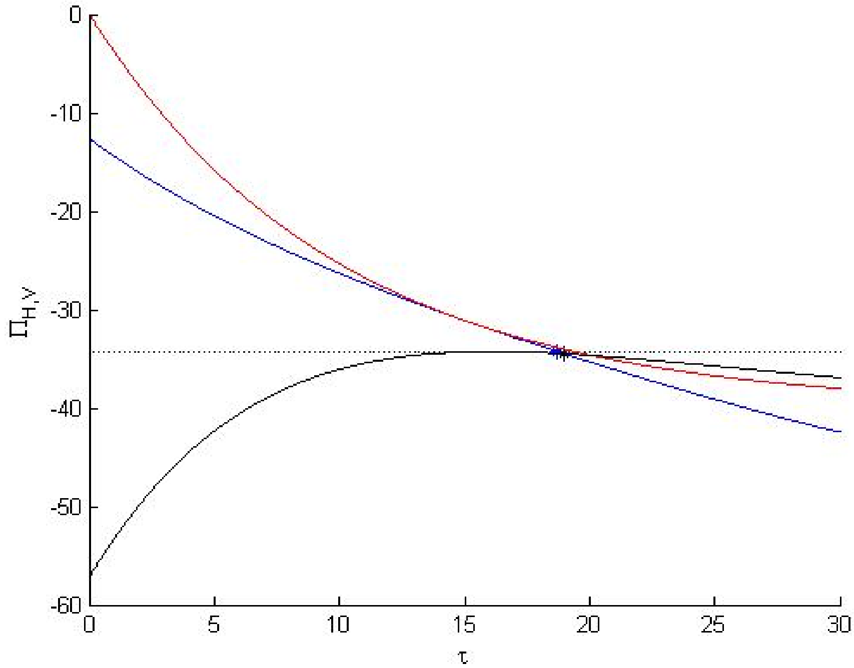

Figure 3.2: Π_V where $z = B_H$ as compared to V's payoff's without z. The dashed black line is the $y = \max(Pi_V(p,\tau)|_{\delta=1})$. This graph was generated using the following values $p = .5$, $q = .3$, $C_P = 15$, $C_T = 5$, $A = 20$, $\lambda = .08$, $c = .4$, $\beta = .1$, $\alpha = .1$, $K = 100$, and $B = 50$.

useful strategy. t_{sell} is always less than the t^* and $\bar{t}$ strategies that we discussed in Chapter 2. However, this is not an issue, because, by setting $z(\tau^\#) = B_H + \varepsilon$, where $1 >> \varepsilon > 0$, V encourages a rational H to accept the buyout. By setting z this way, H is guaranteed to achieve its maximum possible payoff, namely $\max(\Pi_{H,T}) + \varepsilon$, but it is to H's advantage to wait until t_{sell} to accept, thereby giving V a payoff of $\max(\Pi_V)$. It is trivial to show that provided that $t_{sell} > 0$, the buyout strategy is the optimal strategy for V, otherwise the game remains as it was in Chapter 2.

Table 3.1 provides an overview of V's choice of strategies.

3.1.3 Terrorist's Response

It is discussed in [5] that should T become aware of H leaving the coalition, it is in T's interest to "double-cross" H. Such a double cross would involve a large scale highly visible attack. Essentially T would perform a unit level attack with Ap so large that there would be no question that H was supporting them. Such an attack essentially raise the suspicion factor mentioned

Table 3.1: V's Maximin Strategies with Buyout

Order	V's Maximin Strategy	Payoff
$0 \leq t_{sell} \leq t^* \leq \bar{t}$	t_{sell}	$\max(\Pi_V(p,\tau)\|_{\delta=1})$
$0 \leq t_{sell} \leq \bar{t} \leq t^*$	t_{sell}	$\max(\Pi_V(p,\tau)\|_{\delta=1})$
$t_{sell} \leq 0 \leq \bar{t} \leq t^*$	t^*	$-F(p,t^*) - c(p,t^*)$
$t_{sell} \leq 0 \leq t^* \leq \bar{t}$	$\bar{t}$	$-F(p,t^*)$
$t_{sell} \leq \bar{t} \leq 0 \leq t^*$	0	0
$t_{sell} \leq t^* \leq 0 \leq \bar{t}$	0	$(cB-K)/(c+1)$
$t_{sell} \leq \bar{t} \leq t^* \leq 0$	0	0
$t_{sell} \leq t^* \leq \bar{t} \leq 0$	0	0

in Section 2.3 to one, thereby greatly increasing political cost, $O^2(p,\tau)$, of the now broken alliance. Further research could explore how T "double-crossing" H with probability d, and thereby increasing the political costs, effects the buyout price $z(\tau)$.

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CHAPTER 4:
Political Attrition

Assume once again that V can influence C_P by investing z. This time however, V is beginning a slander campaign against H. The longer that V campaigns, the more expensive H's political costs associated with supporting T become. Assume that C_P is now a function of $z(\tau)$, namely $C_P(z(\tau))$. Where $z(\tau)$ is on going investment strategy and $C_P(z(\tau))$ meets the criteria set forth in [6] and shown in Appendix B, namely C_P is increasing and concave down.

4.1 Victim's Payoff

Unlike the previous scenario, we assume that V is continually investing z as opposed to a single payoff. Therefore in order to measure V's payoff we create an investment function $I(\tau)$ to integrate over the interval, similar to the political costs accrued by H as seen in 3.1, therefore $z(\tau) = \int_0^\tau I(s)ds$. V's payoff becomes

$$\Pi_V(p,\tau) = -F^\alpha(p,\tau) - \delta c(p,\tau) - z(\tau) \tag{4.1}$$

$$= -\frac{A(p-q)}{\alpha}(1-\mathrm{e}^{-\alpha\tau}) - \delta\mathrm{e}^{-\alpha\tau}\frac{(K-\mu B)}{1+\mu} - \int_0^\tau I(s)ds. \tag{4.2}$$

We will also apply the following constraints to $I(\tau)$:

$$I(\tau) \geq 0, \quad \forall\tau > 0, \tag{4.3}$$

and

$$I'(\tau) < 0 \tag{4.4}$$

because, as in Chapter 3, we anticipate that as $\tau \to \infty$, V will invest less money as retaliation becomes cheaper.

As is readily apparent in Figure 4.1, this method does not result in the highest payoff for V. This is understandable though because V is not trying to maximize its payoffs, but instead V is attempting to increase H's political costs of being allied with T. V is gambling that the increased political costs will force H to sever ties with T before V loses too much. As a result this technique is only effective if V is powerful enough to pay the consequences.

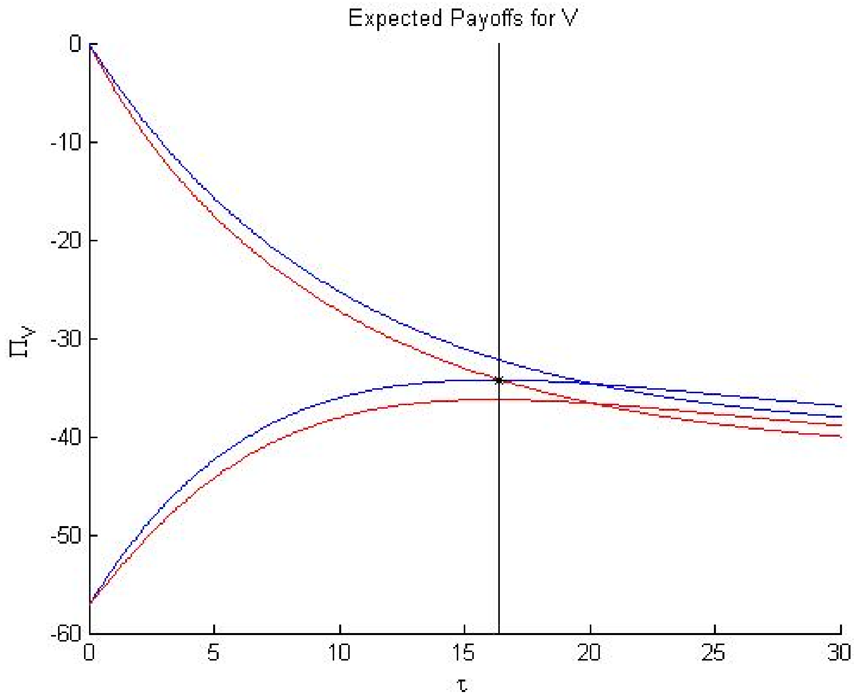

Figure 4.1: In the figure, the blue lines represent Π_V if V does not invest z. The red lines demonstrates Π_V is V does invest, but H remains with T anyway. The black line denotes the time $\tau = \dot{\tau}$ when retaliation has a greater payoff than no retaliation. The values $p = .5$, $q = .3$, $A = 20$, $\lambda = .08$, $c = .4$, $\alpha = .1$, $K = 100$, $B = 50$, and $I(\tau) = \mathrm{e}^{-.5\tau}$ were used in this graph.

Now we can assume that V anticipates acceptable losses while utilizing this method, however, would still like to have a reasonable gain should it succeed. Therefore, we bound z in the following manner,

$$z(\tau) = \int_0^\tau I(s) < \mathrm{e}^{-\alpha\tau}\frac{(K - \mu B)}{1+\mu}. \tag{4.5}$$

Therefore, V is willing to invest, if z is less than the political costs of retaliation. Taking the derivative of 4.5, we find

$$0 < I(\tau) < -\alpha\mathrm{e}^{-\alpha\tau}\frac{(K-\mu B)}{1+\mu} - \mathrm{e}^{-\alpha\tau}\frac{\lambda\mu B}{1+\mu} - \lambda\mu\mathrm{e}^{-\alpha\tau}\frac{(\mu B - K)}{(1+\mu)^2}, \tag{4.6}$$

$$0 < I(\tau) < \mathrm{e}^{-\alpha\tau}\left[(-\alpha - \frac{\lambda\mu}{1+\mu})\frac{(\mu B - K)}{1+\mu} - \frac{\lambda\mu B}{1+\mu}\right]. \tag{4.7}$$

This should ideally occur before the payoff with investment becomes less than the payoff with retaliation which occurs at $\dot{\tau}$. It is important to note that as long as $I(\tau) > 0$, the difference

between $\Pi_V|_{\delta=0}$ and $\Pi_V|_{\delta=0}+z$ will continue to increase as $\tau \to \infty$. However, V can decide to stop investing, set $I=0$, at any time τ_{stop}. If V quits investing, V cannot regain what was already invested and its payoff would be,

$$\Pi_V(p,\tau) = -F^{\alpha}(p,\tau) - \delta c(p,\tau) - z(\tau_{stop}) \tag{4.8}$$

$$= -\frac{A(p-q)}{\alpha}(1-\mathrm{e}^{-\alpha\tau}) - \delta\mathrm{e}^{-\alpha\tau}\frac{(K-\mu B)}{1+\mu} - \int_0^{\tau_{stop}} I(s)ds. \tag{4.9}$$

Thus, we have bounds for z and τ which we can utilize while exploring Θ.

4.2 Influence on Θ

As discussed in Section 2.4, the value of Θ greatly affects the behavior of H. By manipulating z and thereby affecting the political costs, $C_P(z)$, V also impacts Θ, which is now defined as

$$\Theta(z) = \frac{A-C_T}{C_P(z)}. \tag{4.10}$$

We will not assuming any particular shape of $C_P(z)$, but we will assume that it is twice differentiable and that,

$$C_P(z)' > 0 \tag{4.11}$$

and

$$C_P(z)'' < 0. \tag{4.12}$$

Also, we will assume that at $t=0$, H has invested a certain political cost $\bar{C_P}$ that V has no control over, i.e. $C_P(z(0)) = \bar{C_P}$. We make this assumption, because V is attacked at least once before it has a chance to influence H in any way. $\bar{C_P}$ is the political cost H anticipated prior to said first attack. Additionally, assume that without any investment

$$\Theta(0) = \frac{A-C_T}{C_P(0)} > \frac{c}{c+1} \tag{4.13}$$

and

$$\lim_{z\to\infty}\Theta(z) < \frac{c}{c+1}. \tag{4.14}$$

Thus, we need to find the min z such that $\Theta(z) \leq \frac{c}{c+1}$. By rearranging the equation we have

$$C_P(z) \geq \frac{(c+1)(A-C_T)}{c} = \Theta_{break}. \tag{4.15}$$

Thus, the minimum z required to force H to abandon its coalition with T is

$$z(\tau) = C_P^{-1}(\Theta_{break}). \tag{4.16}$$

If V implemented 4.16 for the entire the resultant payoffs for H can be seen in Figure 4.2. Realistically, this is not possible, since $z(0) = 0$, however it does demonstrate the increased costs H has to endure when C_P is increased. Also, the figure depicts the deadly effectiveness of the attrition strategy on weak values of A, in particular $A = 10$ which fails to result in a positive payoff for $\tau < 30$.

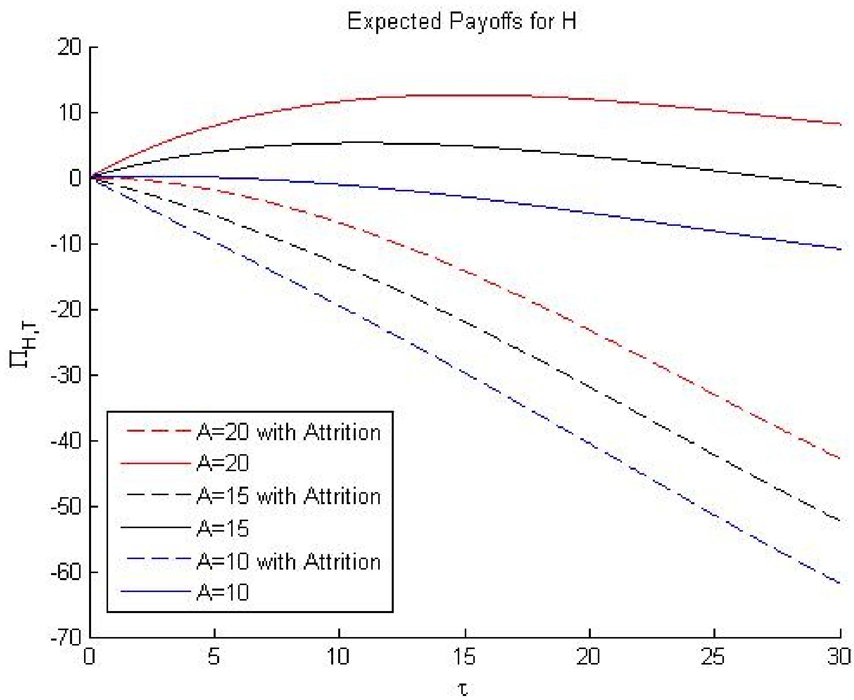

Figure 4.2: This figure displays the expectant payoffs for H, if V is capable of achieving $z = C_P^{-1}(\Theta_{break})$ for the entire game. $q = .3$, $p = .5$, $\lambda = .08$, $c = .4$, $\beta = .1$, $C_T = 5$, and $C_P = 15$ (solid lines) were used in this figure.

4.3 Application

Due to the lack of data to suggest any particular shape of the function C_P, we can not solve for z. However, we can apply criteria for z established in Section 4.3 and the minimum value Θ_{break} where H will end the game to several examples to gain additional understanding of V's implementation of the strategy. Below we will implement examples keeping the following

values constant: $C_T = 5$, $p = .5$, $q = .3$, $\lambda = .08$, $c = .4$, $K = 100$, $B = 50$, $\alpha = .1$ and $\beta = .1$. We will assume that C_P is of the form

$$C_P = \bar{C_P} + z(\tau), \tag{4.17}$$

where $\bar{C_P}$ is the political costs that H anticipates prior to the first attack, we will preset $\bar{C_P} = 15$. By initiating the political costs to this value $A > 9.286$, so we will begin our examples at $A = 10$. Also, we will assume

$$z(\tau) = R \int_0^\tau \mathrm{e}^{-\gamma s} ds, \tag{4.18}$$

where R is the maximum single investment that V is willing to pay and γ is the decay rate.

The assumptions for z and $C_{P(z)}$ fall within our constraints and are therefore valid for exemplary purposes.The tabulated vales below are the results of playing the game for various A. The bold text is the best payoff for V and the strategy employed can be found in the far right column. The payoff values $\Pi_V - z$, $\Pi_H - z$ and Π_H are evaluated at the time t_{break}. The "$E[\Pi_V]$" payoff value represents the payoff the V receives if it follows the strategies in Table 2.1 and "$E[\Pi_H]$" is H's associated payoff. We will modify the values R and γ two times apiece in order to assess the best way to approach the attrition strategy.

For our first example we will utilize $z(\tau)$ with $R = 5$ and $\gamma = .1$ as a base line for comparisons, Table 4.1. Second, will modify increase our decay rate to $\gamma = .2$, which should increase the τ_{break} thereby increasing the length of the integral, Table 4.2. In our third example, we will reduce γ to $-.05$, Table 4.3. Doing so should reduce τ_{break}, and result in higher investments being paid out for a longer time. For example four, we will examine the effects of decreasing R to 2.5, Table 4.4. This will reduce the amount of money invested in any singular investment, and as such, we should require additional investments in order to sum the appropriate z. Lastly, we will observe the effects of increasing $R = 15$, Table 4.5. The high R value in this case should result in higher singluar investments, and as such z should be reached in a more timely manner.

The examples show that the attrition strategy is a viable strategy for small A. As the results in Tables 4.3 and 4.3 demonstrate, by decreasing γ and/or increasing R V can make the method even more effective. Due to the decaying nature of the investments, the most effective strategies involve larger payments in order to reach $z = C_P^{-1}(\Theta_{break})$ as soon as possible. Also, V is

Table 4.1: Table of Values for Example 1

A	Θ_{break}	$\bar{t}$	t^*	τ_{break}	$\Pi_V - z$	$\Pi_H - z$	$E[\Pi_V]$	$E[\Pi_H]$	Strategy
10	17.5	20.117	22.109	.510	**-3.480**	.0093	-17.325	4.393	Attrition
12.5	26.25	20.117	20.609	2.550	**-16.881**	.285	-21.656	8.724	Attrition
15	35.0	20.117	19.129	5.110	-32.008	1.258	**-25.965**	12.760	Engage
20	52.5	20.117	16.160	13.860	-67.493	7.625	**-34.247**	19.738	Engage

Table 4.2: Table of Values for Example 2

A	Θ_{break}	$\bar{t}$	t^*	τ_{break}	$\Pi_V - z$	$\Pi_H - z$	$E[\Pi_V]$	$E[\Pi_H]$	Strategy
10	17.5	20.117	22.109	.530	**-3.547**	.0093	-17.325	4.393	Attrition
12.5	26.25	20.117	20.609	2.990	**-17.713**	.379	-21.656	8.724	Attrition
15	35.0	20.117	19.129	8.050	-36.590	2.527	**-25.965**	12.760	Engage
20	52.5	20.117	16.160	189	-65.0	25.421	**-34.247**	19.738	Engage

Table 4.3: Table of Values for Example 3

A	Θ_{break}	$\bar{t}$	t^*	τ_{break}	$\Pi_V - z$	$\Pi_H - z$	$E[\Pi_V]$	$E[\Pi_H]$	Strategy
10	17.5	20.117	22.109	.510	**-3.512**	.0084	-17.325	4.393	Attrition
12.5	26.25	20.117	20.609	2.39	**-16.578**	.252	-21.656	8.724	Attrition
15	35.0	20.117	19.129	4.46	-30.784	1.009	**-25.965**	12.760	Engage
20	52.5	20.117	16.160	9.4	-61.875	4.707	**-34.247**	19.738	Engage

Table 4.4: Table of Values for Example 4

A	Θ_{break}	$\bar{t}$	t^*	τ_{break}	$\Pi_V - z$	$\Pi_H - z$	$E[\Pi_V]$	$E[\Pi_H]$	Strategy
10	17.5	20.117	22.109	1.05	**-4.485**	.0367	-17.325	4.393	Attrition
12.5	26.25	20.117	20.609	5.98	-22.504	1.213	**-21.656**	8.724	Break-Even
15	35.0	20.117	19.129	16.09	-43.995	5.964	**-25.965**	12.760	Engage
20	52.5	20.117	16.160	>250	-65.00	26.371	**-34.247**	19.738	Engage

experiencing continued attacks, so the more investments that V has to make, the more attacks V must endure. In the absence of sufficient R and γ values, V's best strategy is to avoid investing all together. However, should V decide that it is to V's advantage to accept the losses and reduce H's payoff, V should invest until $\bar{t}$ is achieved, then immediately retaliate and end the game.

Table 4.5: Table of Values for Example 5

A	Θ_{break}	$\bar{t}$	t^*	τ_{break}	$\Pi_V - z$	$\Pi_H - z$	$E[\Pi_V]$	$E[\Pi_H]$	Strategy
10	17.5	20.117	22.109	.170	**-2.866**	.0007	-17.325	4.393	Attrition
12.5	26.25	20.117	20.609	.78	**-13.131**	.0304	-21.656	8.724	Attrition
15	35.0	20.117	19.129	1.43	**-23.984**	.131	-25.965	12.760	Attrition
20	52.5	20.117	16.160	2.88	-47.545	.705	**-34.247**	19.738	Engage

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CHAPTER 5:
Using Intelligence

Through the use of espionage V can provide itself a several additional strategies. Intelligence regarding the terrorist network can help V "harden" itself against T, thereby reducing the effectiveness of A, it can interrupt the network of materials that H is providing, or it can gather evidence to increase the suspicion rate.

5.1 Using Espionage to Enhance the Suspicion Rate

Assume that V can invest into intelligence in an attempt to gain an advantage over H. With this intelligence, V gains the additional evidence that increases the suspicion of a coalition. The increased suspicion in turn will influence the break-even time for retaliation, $\bar{t}$, and break-alliance time for H, $s^{\#}$. In this version of the game, we assume V makes continual investments, i, over the time interval to achieve a total investment, I, at time τ. At any given time, the effectiveness of his espionage is modeled by $f(I,\tau)$, where $f(I,\tau)$ is a probability function that modifies the suspicion rate λ.

Let $i(\tau)$ denote the investment V has allocated for intelligence at a given τ. As the game continues, V will continue to invest $i(\tau)$ until the game is over in order to maintain its spy network. Following the discount factor logic that we have assumed throughout this game, we can assume that i is of the form

$$i(\tau) = C\mathrm{e}^{-\alpha\tau} \tag{5.1}$$

and

$$\int_0^\tau i(s)ds = \frac{C(1-\mathrm{e}^{-\alpha\tau})}{\alpha} = I(\tau). \tag{5.2}$$

Also, we assume that our investment will yield a positive return within the suspicion factor, so $\lim_{I\to\infty} \mu \to \infty$.

In [6] it was shown that λ was derived through the use of continuous Bayesian updating. It was determined that $\ln(\mu)$ was multiplied by $\ln(p/q)$ during a time period with an attack and by $\ln[(1-p)/(1-q)]$ during a period of without an attack. For a T that is allied with H, the attacks occur with a frequency p, in expectation $\ln(\mu)$ increases at our rate λ, 2.6. Thus, the larger the differential between q and p the greater the increase in our suspicions.

Now, it is important to distinguish that V cannot find evidence if H is not supporting T. Recall from 2.6, when T is alone $p = q$, which yields $\lambda = 0$ and the suspicion factor $\mu(\tau) = c$ for all τ. With investment, this should remain the case, i.e. continued investments when $p = q$ yields no return. Also, at time $\tau = 0$, the initial suspicion factor is equal to c and $\int_0^0 i(s)ds = 0$, so $\mu(0) = c$. V expects that following an attack, his intelligence will be able to collect additional evidence of collusion with probability $f(I,\tau)$, but will not recover any data when there is not attack. Thus, λ becomes

$$\dot{\lambda} = (p + f(I,\tau)) \ln\left(\frac{p}{q}\right) + (1-p)\ln\left(\frac{1-p}{1-q}\right). \tag{5.3}$$

Where $f(i)$ has the constraints

$$f'(i) > 0, \tag{5.4}$$

$$f''(i) < 0, \tag{5.5}$$

and

$$\lim_{i \to \infty} f(I,\tau) \to 1. \tag{5.6}$$

Thus, we will assume $f(I,\tau)$ is of the form

$$R(1 - \mathrm{e}^{-\omega I(\tau)\tau}), \tag{5.7}$$

where $0 \le R < 1$ is the evidence collected and $0 < \omega << 1$ is a retarding constant. V's resultant payoff is as follows

$$\Pi_V(p,\tau) = -\frac{A(p-q)}{\alpha}(1-\mathrm{e}^{-\alpha\tau}) - \delta\mathrm{e}^{-\alpha\tau}\frac{(K - \bar{\mu}(\tau,I)B)}{1+\bar{\mu}(\tau,I)} - \frac{C(1-\mathrm{e}^{-\alpha\tau})}{\alpha}, \tag{5.8}$$

$$\Pi_V(p,\tau) = -\frac{(A+C)(p-q)}{\alpha}(1-\mathrm{e}^{-\alpha\tau}) - \delta\mathrm{e}^{-\alpha\tau}\frac{(K - \bar{\mu}(\tau,I)B)}{1+\bar{\mu}(\tau,I)}, \tag{5.9}$$

where $\bar{\mu}$ represents the suspicion factor with the modified $\dot{\lambda}$. H's payoff is modified from 3.1 to

$$\Pi_{H,T}(p,\tau) = \frac{(A-C_T)(p-q)}{\beta}(1-\mathrm{e}^{-\beta\tau}) - \int_0^\tau \frac{C_p\bar{\mu}(v,I)}{\bar{\mu}(v,I)+1}(p-q)\mathrm{e}^{-\beta v}dv - \delta C_R \mathrm{e}^{-\beta\tau}. \tag{5.10}$$

Now we will distinguish a time $\acute{t}$ which represents the break-even point for retaliation during the game with espionage. This break-even point, $\acute{t}$, is found in a similar manner to $\bar{t}$ and, since

$\dot{\lambda} \geq \lambda$,

$$\dot{t} = \frac{\ln(K/cB)}{\dot{\lambda}} \leq \frac{\ln(K/Bc)}{\lambda} = \bar{t}. \tag{5.11}$$

There is also a time $\hat{t}$ that is determined by the positive root $\bar{\mu}$ of the quadratic equation that results from differentiating 5.9 ($\delta = 1$) with respect to τ. Namely,

$$\begin{aligned} 0 = &- [(A+C)(p-q) + \alpha B]\bar{\mu}^2 + [-2(A+C)(p-q) - (\alpha - \dot{\lambda}')B + \\ &(\dot{\lambda}' + \alpha)K]\bar{\mu} + [\alpha K - (A+C)(p-q)]. \end{aligned}$$

Algebraically this number is very difficult to find due to the time dependence of $\dot{\lambda}$, however, through analysis of this quadratic we can gain an understanding of the where the payoff functions maximums and minimums are located. The first term of the quadratic is negative and the final term is usually positive. This implies that there is at least one positive solution $\bar{\mu}$ to the quadratic. The time associated with this value is

$$\hat{t} = \begin{cases} \frac{\ln(\bar{\mu}/c)}{\dot{\lambda}} & \text{if } \bar{\mu} \geq c, \\ 0 & \text{otherwise,} \end{cases} \tag{5.12}$$

but as mentioned previously this is difficult to determine due to the changing nature of λ, however, because $\dot{\lambda} > \lambda$, it can be assumed that $\hat{t} < t^*$. This means that if intelligence is used the maximum value of retaliation occurs earlier than without. Also, we should note that for K small or $A + C$ large the final term of the quadratic is negative. In this case, there is no positive root and the maximizing value would be at $\hat{t} = 0$.

The intended outcome of investment in espionage is an improved suspicion factor. It is apparent in Figure 5.1 and our understanding of $\hat{t}$ and $\bar{t}$ that this is the case. The increased suspicion rate causes the break-even point and the maximum payoff to occur so early. It is possible to shift these values farther to the left by adjusting C but as is apparent in Figure 5.2, the larger the C value the smaller the maximum payoff. This is due to the increase in I, diminishing our returns at higher values of C. Also, as discussed earlier, for excessively high C values the final term of the quadratic can become negative causing our maximum payoff with retaliation to occur at $\tau = 0$.

The use of espionage does not change V's strategies significantly in the game. V's payoffs change, however, the minimax strategies that he employs remain the same, see Table 5.1 below. H's strategies stay the same during this game. The greatest effect that espionage has on H is

during the case when $\frac{c}{c+1} < \Theta < 1$. In this scenarios it is to H's interest to abandon T at $s^{\#}$, which will be pushed to the left just as $\hat{t}$ and $\dot{t}$ were pushed left.

Table 5.1: V's Maximin Strategies During Espionage

Order	V's Maximum Strategy	Payoff
$0 \leq \hat{t} \leq \dot{t}$	$\hat{t}$	$-F(p,\hat{t}) - c(p,\hat{t})$
$0 \leq \dot{t} \leq \hat{t}$	$\dot{t}$	$-F(p,\bar{t})$
$\dot{t} \leq 0 \leq \hat{t}$	0	0
$\hat{t} \leq 0 \leq \dot{t}$	0	$(cB - K)/(c+1)$
$\dot{t} \leq \hat{t} \leq 0$	0	0
$\hat{t} \leq \dot{t} \leq 0$	0	0

5.2 Future Works with Espionage Investments

Espionage could have a wide variety of impacts on this model. Linder proposed through investment V could cause increased prices in material costs, or diminish attack effectiveness by discovering target locations, however, other options present themselves. An interesting future problem might also involve the use of directed force in retaliation. Feasibly, V could accrue intelligence with the purpose of using a directed attack against a terrorist stronghold within H's borders. This action would yield a greater positive benefit B for V, while simultaneously reducing K should the stronghold meet the four requirements outlined in the law of war.

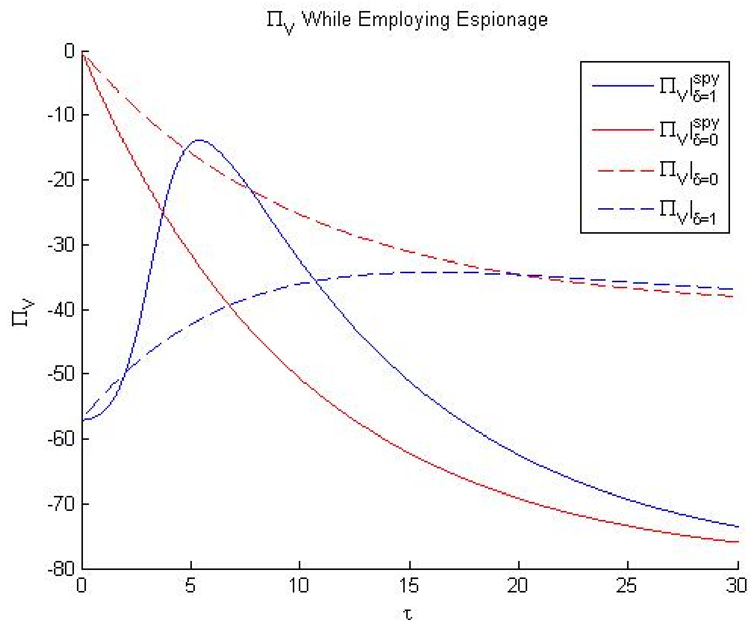

Figure 5.1: This figure displays the expectant payoffs for V while using espionage versus without espionage. In this picture V expects $\Pi_V(p,\hat{t})$ which its the intersection of the two solid colored lines near $\tau = 5$. Without espionage (dashed lines) V expects much less and earns his payoff around $\tau = 18$. $A = 20$, $q = .3$, $p = .5$, $\lambda = .08$ (dashed lines), $c = .4$, $\alpha = .1$, $C = 4$, $B = 50$,$R = .4$, and $K = 100$ were used in this figure.

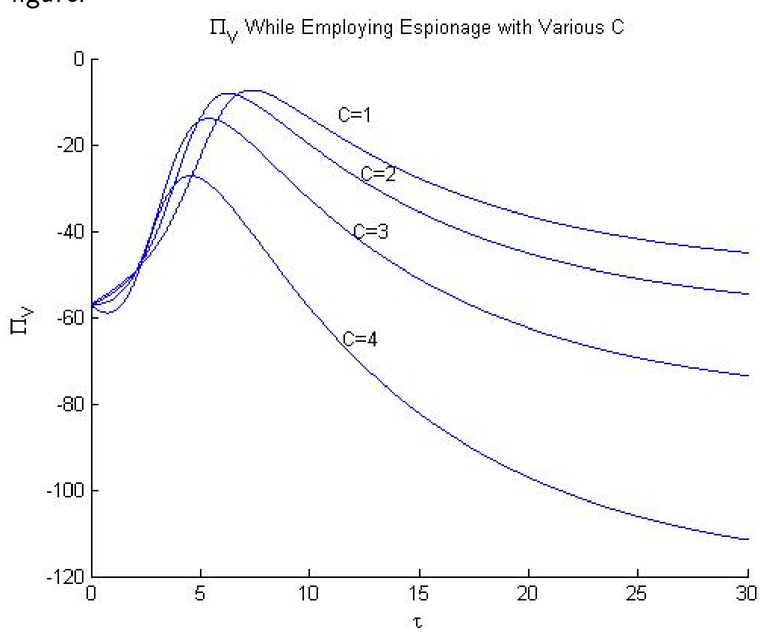

Figure 5.2: This figure displays the expectant payoffs for V while using espionage and varying C. As can be seen, the larger the C value the faster that V reaches his maximum payoff, however, V receives a lower payoff by doing so.$A = 20$, $R = .4$, $q = .3$, $p = .5$, $\lambda = .08$ (dashed lines), $c = .4$, $\alpha = .1$, $B = 50$, and $K = 100$ were used in this figure.

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CHAPTER 6: Conclusion

We did not alter the game's three potential outcomes, but our efforts changed the payoffs significantly. The game still ends when: 1) the state can continue sponsorship until the victim has decided that sufficient evidence of coalition exists and retaliates against the sponsor, for example retaliation at $\tau = 0$, 2) in the case of an imminent retaliatory attack, the sponsor can decide to break off sponsorship, ending the game before he can be retaliated against, $s = t - \varepsilon$, 3) the state sponsor in the face of political costs that exceed the material benefits of sponsorship can abandon the partnership even whether or not retaliation is a threat, $\Theta < \frac{c}{c+1}$.

Our analysis shows that V can improve his payoffs within the game proposed by Lindner, by appropriately investing in areas where H is weak, see Table 6.1 (Note that $\max(\Pi_V) = \Pi_{Buy}$). By carefully considering his investments V can force H to end the game early or arrange the game so that is less in H's favor. Our exploration of the buyout, political attrition and espionage strategies yielded some interesting results. Each method had its own strengths and weaknesses that were caused by their method of exploitation.

By far the most stable of the methods was the buyout method. Assuming that the players were both rational and that $t_{sell} > 0$, it is to both of their benefits to accept the buyout strategy. By the nature of the strategy, if H agrees to V's offer both parties can exceed what they would have achieved acting independently. H will receive more than the maximum achievable profit with T, and V is guaranteed at least the maximum value achievable by retaliating. The added benefit of the strategy lies in the single payout nature of the deal, thus if H refuses V is no worse off.

Political attrition was by far the most costly of the methods, however, this was assumed to be the case from the beginning. The idea was built around the victim's state willingness to "bleed" H, therefore, it is only effective for strong victim states that were willing to accept a loss. The strategy of political attrition relied on the vulnerability of the political cost that H needed to endure while supporting T. By increasing the political cost associated with sponsorship, V was able to reduce the acceptable profit ratio, Θ, until H could no longer support T. For small A values this method was exceptionally effective because C_P did not need to be significantly elevated to diminish Θ, however, for large A, V assumed heavy loses in the effort to achieve his goal. An added benefit of this method was that the H was unable to profit achieve high profit.

Table 6.1: Payoffs Associated with V's Alternate Strategies

A	Π_{Buy}	Π_{Att}	Π_{Spy}
10	-17.81	**-2.866**	-19.2638
15	-25.97	-23.984	**-23.7144**
20	-34.25	-47.545	**-28.0764**
50	-57.14	-237.5033	**-52.3402**

Overall, the method that performed the best was the espionage method. This strategy utilized the ability to increase the suspicion factor, and thereby reduce the cost of retaliation to end the game quickly. Not only did this method end the game quickly, it is able to elevate the expected payoffs of V by a significant margin. The only thing discredit that can be brought to this method, is that it requires retaliation. While the other two methods found ways for V to avoid retaliation costs, this method has no means to force H to abandon T without the threat of retaliation when $\Theta > 1$.

The work in this paper is solely for exploratory use. The functional forms are by no means grounded in empirical data and our parameter suggestions were merely hypothetical. We found no reliable data to support any specific functional forms and thus have utilized simple functions to analyze the data in qualitative term. The purpose of our work is merely to demonstrate new approach to the general problem of resisting sponsored terrorists attacks.

APPENDIX A: Sponsor Choices

Here we will briefly sketch the proof demonstrating H's choice of withdrawal time s as created by Linder [6].

A.1 Optimal s

Foremost, it is never to H's interests to set $s > t$ since retaliation can be avoided by setting $s = t$. So, we will consider the case where $\delta = 0$ and $\tau = s$.

$$\begin{aligned}
\Pi_H(p,s) &= F^{\beta}(p,s) - O^1(p,s) - O^2(p,s) && \text{(A.1)}\\
&= \frac{A(p-q)}{\beta}(1-\mathrm{e}^{-\beta s}) - \frac{C_T(p-q)}{\beta}(1-\mathrm{e}^{-\beta s}) - \int_0^s C_P \frac{\mu(v)}{\mu(v)+1}(p-q)\mathrm{e}^{-\beta v}dv && \text{(A.2)}\\
&= \frac{(A-C_T)(p-q)}{\beta}(1-\mathrm{e}^{-\beta s}) - \int_0^s C_P \frac{\mu(v)}{\mu(v)+1}(p-q)\mathrm{e}^{-\beta v}dv. && \text{(A.3)}
\end{aligned}$$

Is there an incentive to chose $s < t$? Consider the cost $C(s)$ and the benefits $B(s)$ H recieves from his relationship with T.

$$C(s) = \int_0^s C_P \frac{\mu(v)}{\mu(v)+1}(p-q)\mathrm{e}^{-\beta v}dv \quad \text{(A.4)}$$

$$B(s) = \frac{(A-C_T)(p-q)}{\beta}(1-\mathrm{e}^{-\beta s}). \quad \text{(A.5)}$$

Differentiate C and B to get

$$C'(s) = C_P \frac{\mu(v)}{\mu(v)+1}(p-q)\mathrm{e}^{-\beta v} \quad \text{(A.6)}$$

$$B'(s) = (A-C_T)(p-q)(1-\mathrm{e}^{-\beta s}). \quad \text{(A.7)}$$

Then set them equal to one another to get the first-order condition

$$\frac{\mu(s^*)}{\mu(s^*)+1} = \frac{A-C_T}{C_P} = \Theta \quad \text{(A.8)}$$

which can be rewritten as $s^* = \frac{1}{\lambda} \ln \frac{\Theta}{c(1-\Theta)}$.

The second order-order condition $B''(s) - C''(s) < 0$ simplifies to

$$-C_P \frac{\mu'(s)}{(\mu(s)+1)^2} < 0 \tag{A.9}$$

which is always true since $\mu'(s) > 0$ for all $s \geq 0$. Remember $\frac{\mu(s)}{\mu(s)+1}$ is the strictly increasing suspicion factor that bounded above by one and $\frac{\mu(0)}{\mu(0)+1} = \frac{c}{c+1}$. Thus, Θ's bounds are

$$\frac{c}{c+1} \leq \Theta < 1. \tag{A.10}$$

When $\Theta \geq 1$, $B'(s) > C'(s)$ for all s and H's optimal strategy is to continue support until V is about to retaliate, as such, H will set $s^*(t) = t$. When $\Theta < \frac{c}{c+1}$, C' is greater for all s and H will set $s^*(t) = 0$, i.e. sever relations immediately.

For further clarification,

$$s^* = \frac{1}{\lambda} \ln \frac{\Theta}{c(1-\Theta)} = \frac{1}{\lambda} \ln \left[\frac{A - C_T}{c(C_P - (A - C_T))} \right] \tag{A.11}$$

which is meaningful for our Θ conditions, because s^* tends to infinity as Θ approaches one.

APPENDIX B: Investments

Here we will briefly demonstrate that the marginal returns from investing in $C_P(z)$ surpass that of $A(x)$ and $C_t(y)$ as shown in [6]. First assume that V can influence the values A, C_T and C_P by investing x, y and z, respectively. As a result of these investments $\Theta(x,y,z) = \frac{A(x)-C_T(y)}{C_P}$. Without assuming any particular shape of these functions, assume that they are twice differentiable and meet the following criteria:

$$A'(x) < 0 \tag{B.1}$$
$$C_T'(y) > 0 \tag{B.2}$$
$$C_P'(z) > 0 \tag{B.3}$$

and

$$A''(x) > 0 \tag{B.4}$$
$$C_T''(y) < 0 \tag{B.5}$$
$$C_P''(z) < 0. \tag{B.6}$$

Also, we make the assumption that without any investment

$$\Theta(0,0,0) > \frac{c}{c+1} \tag{B.7}$$

and

$$\lim_{x,y,z\to\infty} \Theta(x,y,z) < \frac{c}{c+1}. \tag{B.8}$$

V's optimal investment strategy is obtained by solving the optimization problem

$$\min\ x,y,z \tag{B.9}$$

such that

$$\frac{A(x)-C_T(y)}{C_P(z)} \leq \frac{c}{c+1} \tag{B.10}$$

is given by

$$A'(x) = -C_T'(y) = -\frac{c}{c+1}C_P'(z) \tag{B.11}$$

$$\Theta = \frac{A(x) - C_T(y)}{C_P(z)} = \frac{c}{c+1}. \tag{B.12}$$

Due to the lack of literature and data that would suggest any specific functional form for terms of Θ we will offer a simple example for conceptual understanding.

The relationship between T and H is nearly invisible, which limits V's opportunity to influence this relationship directly. As a result, V's will have more difficulty influencing $C_T(y)$ than $A(x)$. Hence $A(x)$ will be more elastic with respect to investments that $C_T(y)$.

$$\left|\frac{y}{C_T(y)}C_T'(y)\right| < \left|\frac{x}{A(x)}A'(x)\right|. \tag{B.13}$$

Additionally, due to T's ability to strike V in any number of locations, by several various methods, and at any time it desires, V's efforts to reduce A must be done on a large scale across several potential targets. This broad method will undoubtedly be expensive and require a significant investment, x, in order to diminish the effectiveness of T's efforts and H's associated payoff.

Conversely, any investments aimed at increasing the political costs, C_P, that H must endure to maintain its alliance with T, can be more precisely targeted by V. Under these considerations, it is reasonable to assume that $C_P(z)$ is the most elastic of the three targeted areas.

For example from [6], let

$$\begin{aligned} A(x) &= (x+a)^{-2/3} \\ C_T(y) &= y^{1/2} + c_T \\ C_P(z) &= z^{99/100} + c_P \end{aligned}$$

so that for increasing x, y, and z the elasticities approach -2/3, 1/2, and 99/100, respectively.

From the conditions in B.11 and B.12 we get the optimal solutions

$$y^* = \frac{9}{16}(x^* + a)^{\frac{10}{3}} \tag{B.14}$$

$$z^* = \left(\frac{297c}{200(c+1)}\right)^{100} (x^* + a)^{\frac{500}{3}} \tag{B.15}$$

$$z^* = \left(\frac{99c}{50(c+1)}\right)^{100} (y^* + a)^{50} \tag{B.16}$$

The weak curvature of $C_P(z)$ implies that the marginal returns to investment decrease slower than those of $A(x)$ and $C_T(y)$. This is demonstrated by the high exponents in B.15 and B.16.

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APPENDIX C:
Proof I

Assuming that there has been no retaliation and that H is declining V's offer, payoff that H can acquire at a given time τ.

$$\frac{\partial}{\partial \tau}\Pi_{H,T}(p,\tau) = \frac{\partial}{\partial \tau}\left[\frac{(A-C_T)(p-q)}{\beta}(1-\mathrm{e}^{-\beta\tau}) - \int_0^{\tau}\frac{C_P\mu(v)}{\mu(v)+1}(p-q)\mathrm{e}^{-\beta v}dv\right] \tag{C.1}$$

Pull out $(p-q)$ and differentiate,

$$\frac{\partial \Pi_{H,T}}{\partial \tau}(p,\tau) = (p-q)\left[(A-C_T)\mathrm{e}^{-\beta\tau} - \left(\frac{C_P\mu(\tau)}{\mu(\tau)+1}\mathrm{e}^{-\beta\tau}\right)\right]. \tag{C.2}$$

Holding all other variables constant we see that $\frac{\partial \Pi_{H,T}}{\partial \tau}(p,\tau) \to 0$ as $\tau \to \infty$.

If H accepts the offer then

$$\frac{\partial \Pi_{H,V}}{\partial \tau}(p,\tau) = (p-q)\left[(A-C_T)\mathrm{e}^{-\beta\tau} - \left(\frac{C_P\mu(\tau)}{\mu(\tau)+1}\mathrm{e}^{-\beta\tau}\right)\right] + z'(\tau). \tag{C.3}$$

However, we assumed that $z(\tau)$ was a positive decreasing function, thus $z'(\tau) < 0$ and $\frac{\partial \Pi_{H,V}}{\partial \tau} < \frac{\partial \Pi_{H,T}}{\partial \tau}$. From this we can see that $\Pi_{H,V}$ reaches a maximum before $\Pi_{H,T}$. Also, $z(\tau) > 0$, so $\Pi_{H,V} > \Pi_{H,T}$, for all values of τ and p (See Figure C.1). As a result the maximum value of $\Pi_{H,V}$ is greater and occurs before $\Pi_{H,T}$.

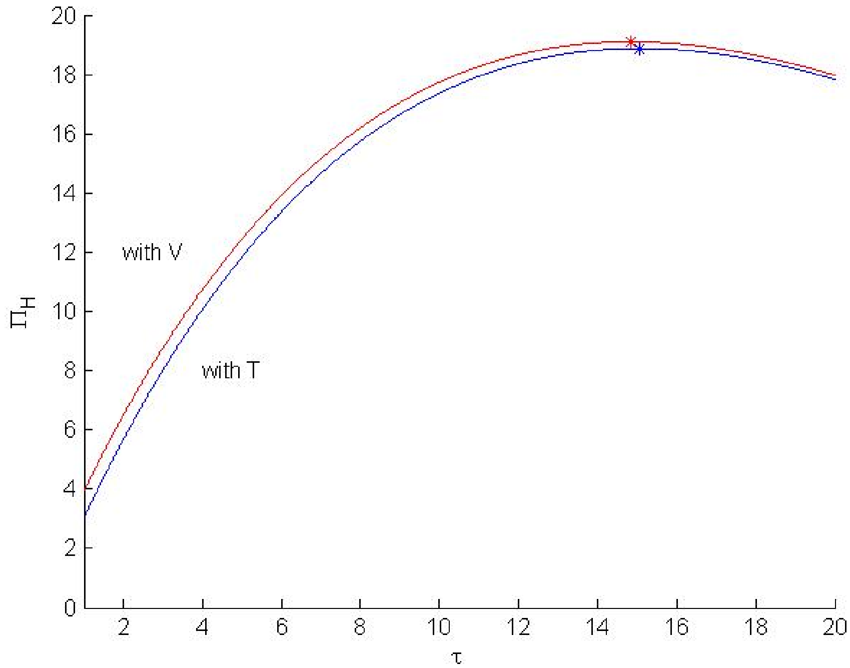

Figure C.1: H can expect greater payoffs from an alliance with V in all cases. The stars above denote the maximum values of $\Pi_{H,V}$ (red) and $\Pi_{H,T}$ (blue). Graph was generated using the following values $p = .6$, $q = .3$, $C_P = 15$, $C_T = 5$, $A = 20$, $\lambda = .08$, $c = .4$, $\beta = .1$, and $z(\tau) = \mathrm{e}^{-.1\tau}$.

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